# Note on Problem 4.4.10c

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I've looked over the textbook publisher's provided solution to problem 4.4.10c from the text, and it looks like they basically just want you to do a whole lot of algebra. Recall that we're trying to optimize the function $A = xy$ subject to the constraint

$$2x + y + \sqrt{x^2 + y^2} = 200.$$

We'd like to solve the constraint equation for either $x$ or $y$ so that we can plug one into our expression for $A$ and make it into a function of one variable. We're eventually going to have to square both sides, so let's rearrange things to minimize the resulting mess:

$$2x + y = 200 - \sqrt{x^2 + y^2}$$

(That step wasn't strictly necessary, but if you try leaving it out you'll appreciate how much work it saves.) Okay, now it's time to do some work:

$$(2x + y)^2 = (200 - \sqrt{x^2 + y^2})^2$$

$$4x^2 + 4xy + y^2 = 40000 - 400\sqrt{x^2 + y^2} + x^2 + y^2$$

Simplify:

$$3x^2 + 4xy + 400\sqrt{x^2 + y^2} = 40000.$$

Now go back to the original constraint equation $2x+y+\sqrt{x^2 + y^2} = 200$, which I'll rewrite as $\sqrt{x^2 + y^2} = 200 - 2x - y$. We can plug this into the equation above, giving

$$3x^2 + 4xy + 400(200 - 2x - y) = 40000,$$

and from here it's not too hard to solve for $y$. You should be able to take things from here.